Lecture 23 - Nov. 28

Bridge Controller

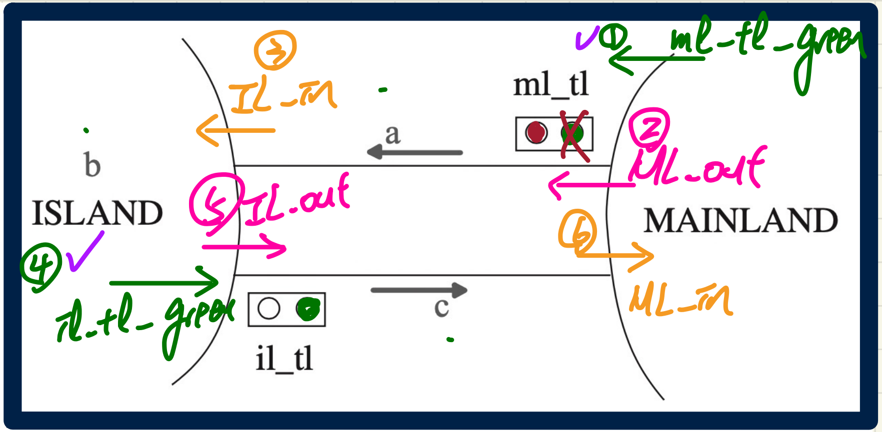
2nd Refinement: Splitting Guards
Adding Invariant to Prove INV

Announcements/Reminders

- **Lab5** released (due on Tuesday, December 3)
- **WrittenTest2** results released
- **Exam** review sessions polling
- Data Sheet:
  + Hand-Writing & Screenshots allowed
  + Font size requirement: $\geq$ 10pt

Bridge Controller: "Old" and "New" Events

Only ① and ④ require safety & capacity checks



Single Car Travel:

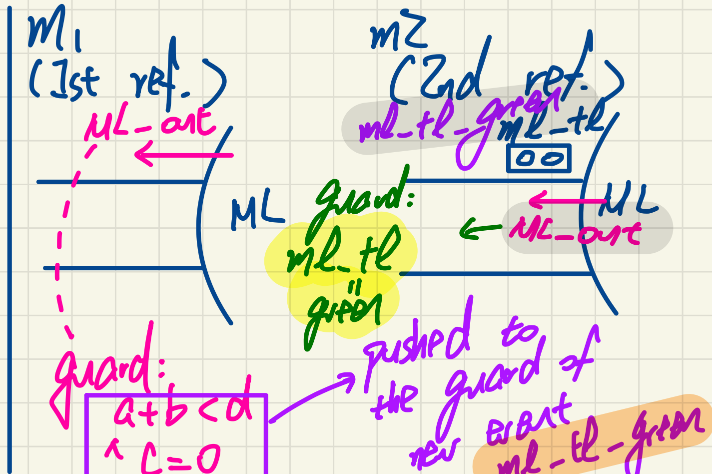
$\boxed{\text{init}}$ → ml_tl = red.
il_tl

⟨init, ①ml_tl_green, ②ML_out, ③IL_in, ④il_tl_green, ⑤IL_out, ⑥ML_in⟩

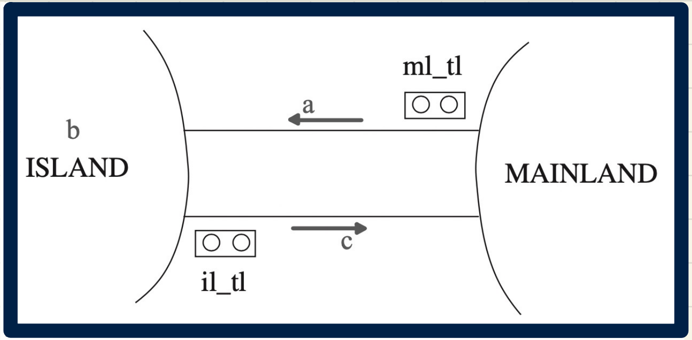
a, b, c are computer variables

↳ $\boxed{\text{drivers}}$ should not access their values

↳ ML_out
IL_out) drivers should only be concerned about traffic lights colours.



Bridge Controller: Guards of "old" Events 2nd Refinement



sets: $COLOR$

constants: $red, green$

axioms:
axm2_1 : $COLOR = \{green, red\}$
axm2_2 : $green \neq red$

variables:
a, b, c
ml_tl
il_tl

invariants:
inv2_1 : $ml\_tl \in COLOUR$
inv2_2 : $il\_tl \in COLOUR$
inv2_3 : $ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
inv2_4 : $il\_tl = green \Rightarrow b > 0 \wedge a = 0$

In order for these two events not to be enabled at the same time, both tl's cannot be green

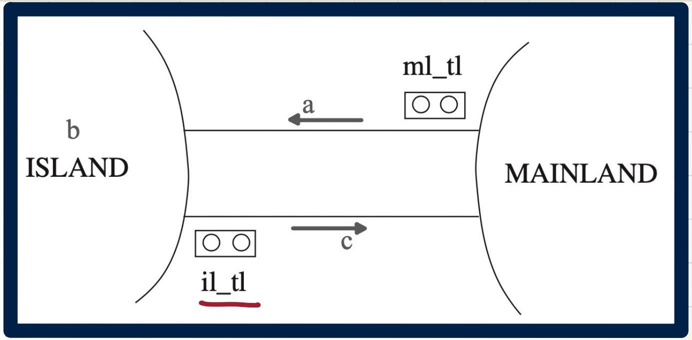
ML_out: A car exits **mainland** (getting onto the **bridge**).

ML_out
when
??   ml_tl = green
then
$a := a + 1$
end

IL_out: A car exits **island** (getting onto the **bridge**).

IL_out
when
??   il_tl = green
then
$b := b - 1$
$c := c + 1$
end

Bridge Controller: Guards of “new” Events 2nd Refinement



sets: *COLOR*

constants: *red*, *green*

axioms:
axm2_1 : $COLOR = \{green, red\}$
axm2_2 : $green \neq red$

variables:
a, b, c
ml_tl
il_tl

invariants:
inv2_1 : $ml\_tl \in COLOUR$
inv2_2 : $il\_tl \in COLOUR$
inv2_3 : $ml\_tl = green \Rightarrow a + b < d \land c = 0$
inv2_4 : $il\_tl = green \Rightarrow b > 0 \land a = 0$

ML_tl_green:

turn the traffic light **ml_tl** to green

```
ML_tl_green
  when
      ??
  then
    ml_tl := green
  end
```

abstract guard of ML_out in m_1

$a + b < d$
$c = 0$

$ml\_tl = red$

IL_tl_green:

turn the traffic light **il_tl** to green

```
IL_tl_green
  when
      ??
  then
    il_tl := green
  end
```

$b > 0$
$a = 0$

abstract guard of IL_out in m_1

$il\_tl = red$

PO/VC Rule of Invariant Preservation: Sequents

Abstract m1

variables: a, b, c

invariants:
- **inv1_1** : $a \in \mathbb{N}$
- **inv1_2** : $b \in \mathbb{N}$
- **inv1_3** : $c \in \mathbb{N}$
- **inv1_4** : $a + b + c = n$
- **inv1_5** : $a = 0 \lor c = 0$

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

IL_out
when
$b > 0$
$a = 0$
then
$b := b - 1$
$c := c + 1$
end

$$A(c)$$
$$I(c, v)$$
$$J(c, v, w)$$
$$H(c, w)$$
$$\vdash$$
$$J_i(c, E(c, v), F(c, w))$$

Concrete m2

variables:
a, b, c
ml_tl
il_tl

invariants:
- **inv2_1** : $ml\_tl \in COLOUR$
- **inv2_2** : $il\_tl \in COLOUR$
- **inv2_3** : $ml\_tl = green \Rightarrow a + b < d \land c = 0$
- **inv2_4** : $il\_tl = green \Rightarrow b > 0 \land \boxed{a} = 0$

ML_out
when
$ml\_tl = green$
then
$a := a + 1$
end

IL_out
when
$il\_tl = green$
then
$b := b - 1$
$c := c + 1$
end

ML_out/inv2_4/INV

axm0_1	$\{\ d \in \mathbb{N}$
axm0_2	$\{\ d > 0$
axm2_1	$\{\ COLOUR = \{green, red\}$
axm2_2	$\{\ green \neq red$
inv0_1	$\{\ n \in \mathbb{N}$
inv0_2	$\{\ n \leq d$
inv1_1	$\{\ a \in \mathbb{N}$
inv1_2	$\{\ b \in \mathbb{N}$
inv1_3	$\{\ c \in \mathbb{N}$
inv1_4	$\{\ a + b + c = n$
inv1_5	$\{\ a = 0 \lor c = 0$
inv2_1	$\{\ ml\_tl \in COLOUR$
inv2_2	$\{\ il\_tl \in COLOUR$
inv2_3	$\{\ ml\_tl = green \Rightarrow a + b < d \land c = 0$
inv2_4	$\{\ il\_tl = green \Rightarrow b > 0 \land a = 0$
Concrete guards of *ML_out*	$\{\ ml\_tl = green$
	$\vdash$
Concrete invariant **inv2_4** with *ML_out*'s effect in the post-state	$\{\ il\_tl = green \Rightarrow b > 0 \land \boxed{(a + 1)} = 0$

Exercise: Specify IL_out/inv2_3/INV

Example Inference Rules

$$\frac{H, P, Q \vdash R}{H, P, P \Rightarrow Q \vdash R} \quad \textbf{IMP\_L}$$

Modus Ponens

$$(p \wedge (p \Rightarrow q)) \Rightarrow q$$

$$\frac{\boxed{H, P} \vdash Q}{H \vdash P \Rightarrow Q} \quad \textbf{IMP\_R}$$

$$H \wedge p \Rightarrow q$$

$$\equiv$$

$$H \Rightarrow (p \Rightarrow q)$$

$$\frac{H, \neg Q \vdash P}{H, \boxed{\neg P \vdash Q}} \quad \textbf{NOT\_L}$$

Contra-positive

$$\neg p \Rightarrow q$$

$$= \{\text{contra-pos}\} \quad \neg q \Rightarrow \boxed{\neg(\neg p)}$$

$$p$$

Discharging POs of m2: Invariant Preservation

First Attempt

$$
\begin{array}{l}
d \in \mathbb{N} \\
d > 0 \\
COLOUR = \{green, red\} \\
green \neq red \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
ml\_tl \in COLOUR \\
il\_tl \in COLOUR \\
ml\_tl = green \Rightarrow a + b < d \land c = 0 \\
il\_tl = green \Rightarrow b > 0 \land a = 0 \\
ml\_tl = green \\
\vdash \\
il\_tl = green \Rightarrow b > 0 \land (a + 1) = 0
\end{array}
$$

ML_out/inv2_4/INV

Outstanding/Unprovable Sequent

$$
\begin{array}{l}
green \neq red \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
1 = 0
\end{array}
$$

True $\Rightarrow$ False = False) Contradiction.

$$\frac{H \vdash P \qquad H \vdash Q}{H \vdash P \land Q}\ \textbf{AND\_R} \qquad \frac{H, P, Q \vdash R}{H, P \land Q \vdash R}\ \textbf{AND\_L}$$

$$\frac{H, P, Q \vdash R}{H, P, P \Rightarrow Q \vdash R}\ \textbf{IMP\_L} \qquad \frac{H, P \vdash Q}{H \vdash P \Rightarrow Q}\ \textbf{IMP\_R}$$

MON

$$
\begin{array}{l}
green \neq red \\
il\_tl = green \Rightarrow b > 0 \land a = 0 \\
ml\_tl = green \\
\vdash \\
\boxed{il\_tl = green} \Rightarrow b > 0 \land (a + 1) = 0
\end{array}
\quad \textbf{IMP\_R} \quad
\begin{array}{l}
green \neq red \\
il\_tl = green \Rightarrow \boxed{b > 0 \land a = 0} \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
b > 0 \land (a + 1) = 0
\end{array}
\quad \textbf{IMP\_L} \quad
\begin{array}{l}
green \neq red \\
\boxed{b > 0 \land a = 0} \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
b > 0 \land (a + 1) = 0
\end{array}
\quad \textbf{AND\_L} \quad
\begin{array}{l}
green \neq red \\
b > 0 \\
a = 0 \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
b > 0 \land (a + 1) = 0
\end{array}
\quad \textbf{AND\_R}
$$

$$
\left\{
\begin{array}{l}
\begin{array}{l}
green \neq red \\
b > 0 \\
a = 0 \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
b > 0
\end{array}
\quad \textbf{HYP} \\
\\
\begin{array}{l}
green \neq red \\
b > 0 \\
a = 0 \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
(a + 1) = 0
\end{array}
\quad \textbf{EQ\_LR, MON} \quad
\begin{array}{l}
green \neq red \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
(0 + 1) = 0
\end{array}
\quad \textbf{ARI} \quad
\begin{array}{l}
green \neq red \\
ml\_tl = green \\
il\_tl = green \\
\vdash \\
1 = 0
\end{array}
\quad ??
\end{array}
\right.
$$

Discharging POs of m2: Invariant Preservation

First Attempt

IL_out/inv2_3/INV

$$\frac{H \vdash P \qquad H \vdash Q}{H \vdash P \wedge Q}\ \textbf{AND\_R} \qquad \frac{H, P, Q \vdash R}{H, P \wedge Q \vdash R}\ \textbf{AND\_L}$$

$$\frac{H, P, Q \vdash R}{H, P, P \Rightarrow Q \vdash R}\ \textbf{IMP\_L} \qquad \frac{H, P \vdash Q}{H \vdash P \Rightarrow Q}\ \textbf{IMP\_R}$$

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \vee c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
$il\_tl = green \Rightarrow b > 0 \wedge a = 0$
$il\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow a + (b-1) < d \wedge (c+1) = 0$

MON

$green \neq red$
$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
$il\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow a + (b-1) < d \wedge (c+1) = 0$

IMP_R

$green \neq red$
$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \wedge (c+1) = 0$

IMP_L

$green \neq red$
$a + b < d \wedge c = 0$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \wedge (c+1) = 0$

AND_L

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \wedge (c+1) = 0$

AND_R

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d$

MON

$a + b < d$
$\vdash$
$a + (b-1) < d$

ARI

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$(c+1) = 0$

EQ_LR, MON

$green \neq red$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$(0 + 1) = 0$

ARI

$green \neq red$
$il\_tl = green$
$ml\_tl = green$
$\vdash$
$1 = 0$

??

exercise!

Fixed

Understanding the Failed Proof on INV

variables:
a, b, c
ml_tl
il_tl

invariants:
inv2_1 : $ml\_tl \in COLOUR$
inv2_2 : $il\_tl \in COLOUR$
inv2_3 : $ml\_tl = green \Rightarrow a + b < d \land c = 0$
inv2_4 : $il\_tl = green \Rightarrow b > 0 \land a = 0$

ML_out
  when
    $ml\_tl = green$
  then
    $a := a + 1$
  end

IL_out
  when
    $il\_tl = green$
  then
    $b := b - 1$
    $c := c + 1$
  end

IL_out/inv2_3/INV

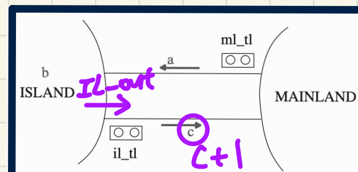
$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land \boxed{c = 0}$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$il\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow a + (b - 1) < d \land (c + 1) = 0$

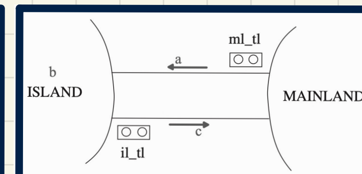
$c = 0 \land c = 1 \equiv \boxed{false}$

$c = 0$ contradicted!

ML_out/inv2_4/INV

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = green$
$\vdash$
$il\_tl = green \Rightarrow b > 0 \land (a + 1) = 0$





Unprovable Sequent:

$$
\begin{array}{ll}
 & green \neq red \\
\land & il\_tl = green \\
\land & ml\_tl = green \\
\vdash & \\
 & 1 = 0
\end{array}
$$

⟨ init	ML_tl_green	ML_out	IL_in	IL_tl_green	$\boxed{\text{IL\_out}}$	ML_out ⟩
$d = 2$	$d = 2$	$d = 2$	$d = 2$	$d = 2$	$d = 2$	$d = 2$
$a' = 0$	$a' = 0$	$\mathbf{a' = 1}$	$\mathbf{a' = 0}$	$a' = 0$	$a' = 0$	$\mathbf{a' = 1}$
$b' = 0$	$b' = 0$	$b' = 0$	$\mathbf{b' = 1}$	$b' = 1$	$\mathbf{b' = 0}$	$b' = 0$
$c' = 0$	$c' = 0$	$c' = 0$	$c' = 0$	$c' = 0$	$\boxed{\mathbf{c' = 1}}$	$c' = 1$
$ml\_tl' = red$	$\mathbf{ml\_tl' = green}$	$ml\_tl' = green$	$ml\_tl' = green$	$ml\_tl' = green$	$ml\_tl' = green$	$ml\_tl' = green$
$il\_tl' = red$	$il\_tl' = red$	$il\_tl' = red$	$il\_tl' = red$	$\mathbf{il\_tl' = green}$	$il\_tl' = green$	$il\_tl' = green$

possible state

Fixing m2: Adding an Invariant

Abstract m1

variables: a, b, c

invariants:
- **inv1_1** : $a \in \mathbb{N}$
- **inv1_2** : $b \in \mathbb{N}$
- **inv1_3** : $c \in \mathbb{N}$
- **inv1_4** : $a + b + c = n$
- **inv1_5** : $a = 0 \lor c = 0$

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

IL_out
when
$b > 0$
$a = 0$
then
$b := b - 1$
$c := c + 1$
end

REQ3	The bridge is one-way or the other, not both at the same time.

inv2_5: $ml\_tl = red \lor il\_tl = red$

Concrete m2

variables:
a, b, c
ml_tl
il_tl

invariants:
- **inv2_1** : $ml\_tl \in COLOUR$
- **inv2_2** : $il\_tl \in COLOUR$
- **inv2_3** : $ml\_tl = green \Rightarrow a + b < d \land c = 0$
- **inv2_4** : $il\_tl = green \Rightarrow b > 0 \land a = 0$

ML_out
when
$ml\_tl = green$
then
$a := a + 1$
end

IL_out
when
$il\_tl = green$
then
$b := b - 1$
$c := c + 1$
end

ML_out/inv2_4/INV

axm0_1	$\{\ d \in \mathbb{N}$
axm0_2	$\{\ d > 0$
axm2_1	$\{\ COLOUR = \{green, red\}$
axm2_2	$\{\ green \neq red$
inv0_1	$\{\ n \in \mathbb{N}$
inv0_2	$\{\ n \leq d$
inv1_1	$\{\ a \in \mathbb{N}$
inv1_2	$\{\ b \in \mathbb{N}$
inv1_3	$\{\ c \in \mathbb{N}$
inv1_4	$\{\ a + b + c = n$
inv1_5	$\{\ a = 0 \lor c = 0$
inv2_1	$\{\ ml\_tl \in COLOUR$
inv2_2	$\{\ il\_tl \in COLOUR$
inv2_3	$\{\ ml\_tl = green \Rightarrow a + b < d \land c = 0$
inv2_4	$\{\ il\_tl = green \Rightarrow b > 0 \land a = 0$
$\boxed{\textbf{inv2\_5}}$	$\{\ ml\_tl = red \lor il\_tl = red$
Concrete guards of *ML_out*	$\{\ ml\_tl = green$
	$\vdash$
Concrete invariant **inv2_4** with *ML_out*'s effect in the post-state	$\{\ il\_tl = green \Rightarrow b > 0 \land (a + 1) = 0$

only different compared with the PO without the change.

Exercise: Specify IL_out/inv2_3/INV

Discharging POs of m2: Invariant Preservation

Second Attempt

ML_out/inv2_4/INV

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$il\_tl = green \Rightarrow b > 0 \land (a + 1) = 0$

MON

$green \neq red$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$il\_tl = green \Rightarrow b > 0 \land (a + 1) = 0$

IMP_R

$green \neq red$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$b > 0 \land (a + 1) = 0$

IMP_L

$green \neq red$
$b > 0 \land a = 0$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$b > 0 \land (a + 1) = 0$

AND_L

$green \neq red$
$b > 0$
$a = 0$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$b > 0 \land (a + 1) = 0$

AND_R

$green \neq red$
$b > 0$
$a = 0$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$b > 0$

HYP

$green \neq red$
$b > 0$
$a = 0$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$(a + 1) = 0$

EQ_LR, MON

$green \neq red$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$(0 + 1) = 0$

ARI

$green \neq red$
$ml\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$1 = 0$

green ≠ red
ml_tl = green
ml_tl = red ∨ il_tl = red
il_tl = green
⊢
1 = 0

OR_L

$green \neq red$ ✗
$\boxed{ml\_tl = green}$
$ml\_tl = red$
$il\_tl = green$
$\vdash$
$1 = 0$

EQ_LR, MON

$green \neq red$
$\boxed{green = red}$
$il\_tl = green$
$\vdash$
$1 = 0$

Approach 1: NOT_L

Approach 2: $green = red$ "false"

exercise

$$\frac{H, \neg Q \vdash P}{H, \neg P \vdash Q} \quad \textbf{NOT\_L}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)} \quad \textbf{EQ\_LR}$$

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \lor Q \vdash R} \quad \textbf{OR\_L}$$

Discharging POs of m2: Invariant Preservation

Second Attempt

IL_out/inv2_3/INV

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow a + (b-1) < d \land (c+1) = 0$

MON

$green \neq red$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$ml\_tl = red \lor il\_tl = red$
$il\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow a + (b-1) < d \land (c+1) = 0$

IMP_R

$green \neq red$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \land (c+1) = 0$

IMP_L

$green \neq red$
$a + b < d \land c = 0$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \land (c+1) = 0$

AND_L

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d \land (c+1) = 0$

AND_R

Branch 1:

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$a + (b-1) < d$

MON

$a + b < d$
$\vdash$
$a + (b-1) < d$

ARI

Branch 2:

$green \neq red$
$a + b < d$
$c = 0$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$(c+1) = 0$

EQ_LR, MON

$green \neq red$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$(0+1) = 0$

ARI

$green \neq red$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$1 = 0$

$green \neq red$
$il\_tl = green$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$1 = 0$

Assignment

$$\frac{H, \neg Q \vdash P}{H, \neg P \vdash Q}\ \textbf{NOT\_L}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \textbf{EQ\_LR}$$

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \lor Q \vdash R}\ \textbf{OR\_L}$$